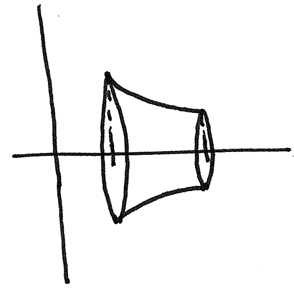
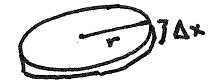
hw set 14
sect 6.2
p 457

1. $y = 1/x$, $x=1$, $x=2$, $y=0$; about x-axis

$$A(x) = r^2\pi = \left(\tfrac{1}{x}\right)^2\pi = \pi/x^2$$

$$V = \int_1^2 \pi/x^2 \, dx = \pi \int_1^2 x^{-2}\,dx$$
$$= \pi\left[-x^{-1}\right]_1^2 = \pi\left[-1/2 + 1\right] = \pi/2$$



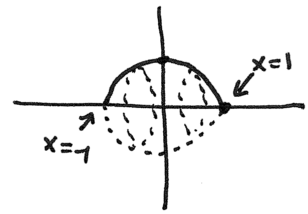


$r = 1/x$

2. $y = 1-x^2$, $y=0$; about x-axis

$$A(x) = r^2\pi = (1-x^2)^2\pi = (1-2x^2+x^4)\pi$$

$$V = \int_{-1}^1 (1-2x^2+x^4)\pi \, dx = \pi \int_{-1}^1 1-2x^2+x^4 \, dx$$
$$= \pi\left[x - \tfrac{2}{3}x^3 + x^5/5\right]_{-1}^1 = \pi\left[(1-2/3+1/5)-(-1+2/3-1/5)\right] = \frac{16\pi}{15}$$

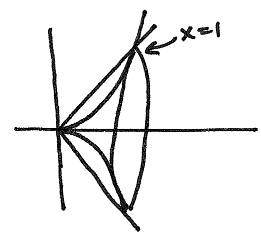


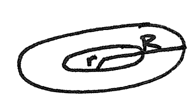
$r = 1-x^2$

5. $y = x^3$, $y=x$, $x \geq 0$; about x-axis

$$A(x) = (R^2-r^2)\pi = (x^2-x^6)\pi$$

$$V = \int_0^1 (x^2-x^6)\pi\,dx = \pi\left[\frac{x^3}{3} - \frac{x^7}{7}\right]_0^1$$
$$= \pi\left[1/3 - 1/7\right] = \frac{4\pi}{21}$$



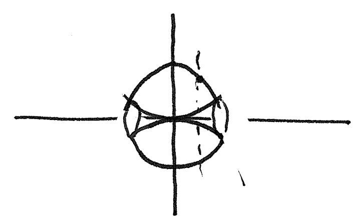


$R = x$
$r = x^3$

6. $y = \frac{1}{4}x^2$, $y = 5-x^2$; about x-axis

$$A(x) = R^2\pi - r^2\pi = (R^2-r^2)\pi = \left[(5-x^2)^2 - \frac{x^4}{16}\right]\pi$$
$$= \left[25 - 10x^2 + x^4 - \frac{x^4}{16}\right]\pi$$

$$V = \int_{-2}^2 \left(25 - 10x^2 + \tfrac{15}{16}x^4\right)\pi\,dx$$
$$= 2\pi \int_0^2 \left(25-10x^2+\tfrac{15}{16}x^4\right)dx$$
$$= 2\pi\left[25x - \tfrac{10}{3}x^3 + \tfrac{3}{16}x^5\right]_0^2 = 2\pi\left(50 - \tfrac{80}{3} + 6\right) = \frac{176\pi}{3}$$



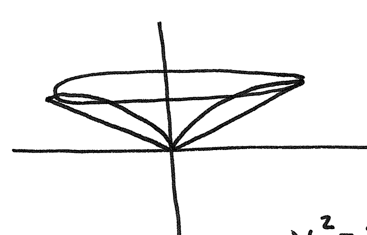
$R = 5-x^2$
$r = \frac{x^2}{4}$

$\frac{x^2}{4} = 5 - x^2$
$x^2 = 20 - 4x^2$
$5x^2 = 20$
$x^2 = 4$, $x = \pm 2$

7. $y^2 = x$, $x = 2y$; about y-axis

$$A(y) = (R^2-r^2)\pi = (4y^2 - y^4)\pi$$

$$V = \int_0^2 (4y^2-y^4)\pi\,dy = \pi\int_0^2 4y^2-y^4\,dy$$
$$= \pi\left[\tfrac{4}{3}y^3 - \tfrac{1}{5}y^5\right]_0^2$$
$$= \pi\left(\tfrac{32}{3} - \tfrac{32}{5}\right) = 32\pi\left(\tfrac{2}{15}\right)$$
$$= \frac{64\pi}{15}$$

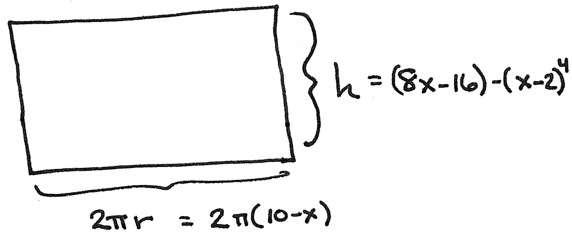


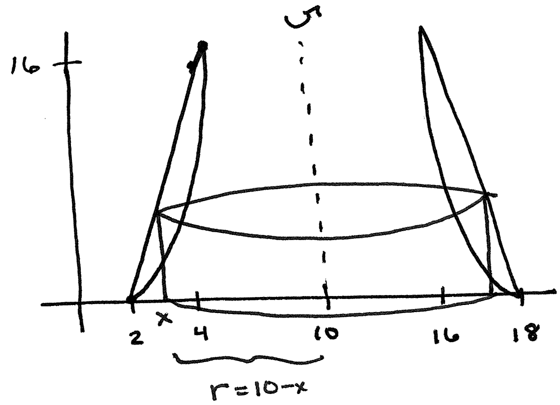
R: $x = 2y$
r: $x = y^2$

$y^2 = 2y$
$y^2 - 2y = 0$
$y(y-2) = 0$
$y = 0, 2$

16. $y = (x-2)^4$

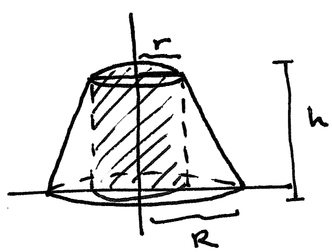
$8x - y = 16 \longrightarrow y = 8x - 16$



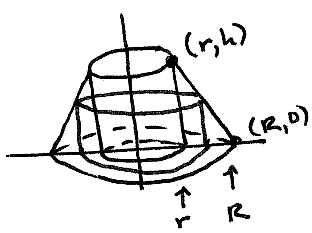


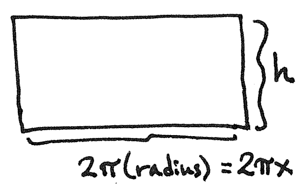
$$V = 2\pi \int_2^4 (10-x)\left[(8x-16)-(x-2)^4\right] dx$$

26.



we know $= \pi r^2 h$ is inner ~~[illegible]~~ cylinder





need an $h(x)$ which has (r,h) & $(R,0)$

$$m = \frac{h-0}{r-R} = \frac{h}{r-R}$$

So $y - y_1 = m(x - x_1)$

$$y - 0 = \frac{h}{r-R}(x-R)$$

$$y = \frac{hx}{r-R} - \frac{hR}{r-R}$$

$$\therefore h(x) = \frac{hx}{r-R} - \frac{hR}{r-R} = h\left(\frac{x-R}{r-R}\right)$$

$$V_o = 2\pi h \int_r^R x\left(\frac{x-R}{r-R}\right) dx$$

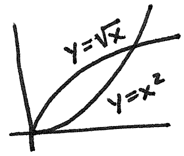
$$= \frac{2\pi h}{r-R}\int_r^R x^2 - Rx \, dx = \frac{2\pi h}{r-R}\left[\frac{x^3}{3} - \frac{Rx^2}{2}\right]_r^R$$

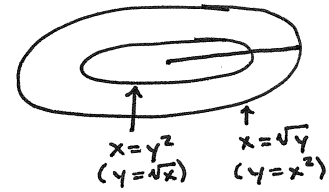
$$= \frac{2\pi h}{r-R}\left[\left(\frac{R^3}{3} - \frac{R^3}{2}\right) - \left(\frac{r^3}{3} - \frac{Rr^2}{2}\right)\right] = \frac{2\pi h}{r-R}\left[-\frac{R^3}{6} - \frac{2r^3}{6} + \frac{3Rr^2}{6}\right]$$

$$= \frac{\pi h}{3}\left[\frac{R^2 + rR - 2r^2}{1}\right]$$

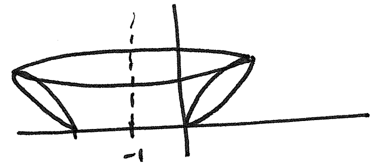
total $V = V_o + \pi r^2 h = \frac{\pi h}{3}\left[R^2 + rR + r^2\right]$

11. $y=x^2$, $x=y^2$; about $x=-1$





$$A(y)=(R^2-r^2)\pi=\left[(1+\sqrt{y})^2-(1+y^2)^2\right]\pi$$

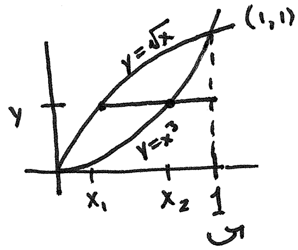


$$V=\pi\int_0^1 (2\sqrt{y}+y-2y^2-y^4)\,dy$$

$$=\pi\left[\tfrac{4}{3}y^{3/2}+\tfrac{1}{2}y^2-\tfrac{2}{3}y^3-\tfrac{1}{5}y^5\right]_0^1$$

$$=\pi\left(4/3+1/2-2/3-1/5\right)=\frac{29\pi}{30}$$

13. $y=x^3$, $y=\sqrt{x}$; about $x=1$



$R=1-x_1=1-y^2$

$r=1-x_2=1-y^{1/3}$

$$A(y)=(R^2-r^2)\pi=\left[(1-y^2)^2-(1-y^{1/3})^2\right]\pi$$

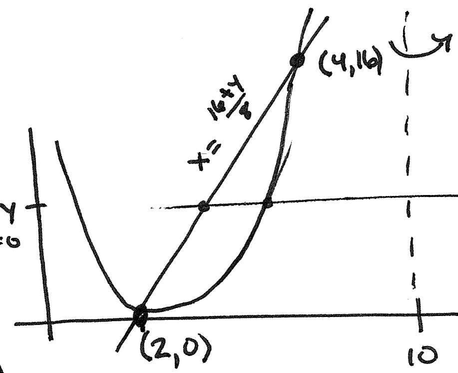
$$=\left[-2y^2+y^4+2y^{1/3}-y^{2/3}\right]\pi$$

$$V=\pi\int_0^1 -2y^2+y^4+2y^{1/3}-y^{2/3}\,dy=\pi\left[-\tfrac{2}{3}y^3+\tfrac{1}{5}y^5+\tfrac{3}{2}y^{4/3}-\tfrac{3}{5}y^{5/3}\right]_0^1$$

$$=\pi\left(-2/3+1/5+3/2-3/5\right)=13\pi/30$$

16. $y=(x-2)^4$, $8x-y=16$; about $x=10$

$y^{1/4}+2=x$ $\qquad$ $y=8x-16$



$(x-2)^4=8x-16$

$-(x-4)(x-2)(x^2+2x+4)=0$

$x=2,4$

$$A(y)=(R^2-r^2)\pi$$

$$=\left[(8+1/8\,y)^2-(12-y^{1/4})^2\right]\pi$$

$$V=\int_0^{16}\left[(8-1/8\,y)^2-(8-y^{1/4})^2\right]\pi\,dy=\left(\frac{1984\pi}{15}\right)$$

answer

$R=10-x_1=-(2+\tfrac{1}{8}y)+10$

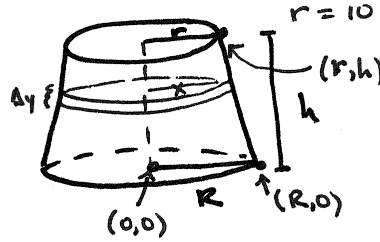
$=8-1/8\,y$

$r=10-x_2=10-(y^{1/4}+2)$

$=8-y^{1/4}$

26. EC

$(r,h),(R,0)$ give $\Rightarrow m=\dfrac{h-0}{r-R}=\dfrac{h}{r-R}$



$$y-0=\frac{h}{r-R}(x-R)$$

$$y=\frac{h}{r-R}x-\frac{Rh}{r-R}$$

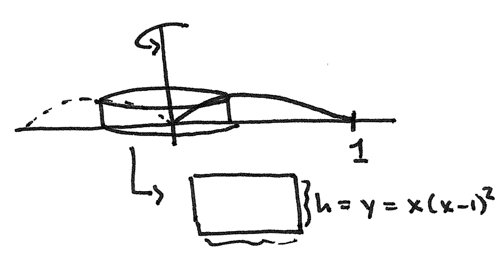
$$x=\frac{r-R}{h}\left[y+\frac{Rh}{r-R}\right]=\frac{r-R}{h}y+R$$

$$A(y)=r^2\pi=x^2\pi=\left(\frac{r-R}{h}y+R\right)^2\pi=\left[\frac{(r-R)^2}{h^2}y^2+\frac{2R(r-R)}{h}y+R^2\right]\pi$$

$$V=\pi\int_0^h \frac{(r-R)^2}{h^2}y^2+\frac{2R(r-R)}{h}y+R^2\,dy=\pi\left[\frac{(r-R)^2y^3}{h^2\cdot 3}+\frac{R(r-R)}{h}y^2+R^2y\right]_0^h=\pi\left[\tfrac{1}{3}(r-R)^2h+R(r-R)h+R^2h\right]$$

$$=\pi\left[\tfrac{1}{3}(r^2-2rR+R^2)h+rRh-\cancel{R^2h}+\cancel{R^2h}\right]=\tfrac{1}{3}h\pi(r^2+rR+R^2)$$

49. $y=x(x-1)^2$, $y=0$; about y-axis



$$V=\int_0^1 2\pi x^2(x-1)^2\,dx$$

$$=2\pi\int_0^1 x^4-2x^3+x^2\,dx$$

$$=2\pi\left[\frac{x^5}{5}-\tfrac{1}{2}x^4+\tfrac{1}{3}x^3\right]_0^1$$

$$=2\pi\left(\tfrac{1}{5}-1/2+1/3\right)=\pi/15$$

Cylinder: $2\pi x^2(x-1)^2$ on $[0,1]$